PATNA UNIVERSITY

COURSE OF STUDY

M.A/M.Sc, MATHEMATICS

SEMESTER- I, II, III & IV

Syllabus of M.A/M.Sc (Mathematics) Semester I

PAPER I (M-101)

Abstract Algebra I, Graph Theory and Lattice Theory

Abstract Algebra I

Unit 1 : Normal and subnormal series composition series of a group, Jordan- Holder Theorem, Solvable groups, commutator subgroup of a group, Nilpotent groups.

Unit 2 : Extension fields , algebraic and transcendental extension, splitting field of Polynomial, separable and inseparable extension, normal extension, constructible real numbers.

Unit 3 : Group of automorphism of a field , fixed field, Galois extensions, Fundamental theorem of Galois theory.

Graph Theory

Unit 4 : Definition of graphs , paths, circuits and subgraphs, induced subgraphs, degree of a vertex, connectivity, planar graphs and their properties, Trees and simple applications of graphs.

Lattice Theory

Unit 5 : Lattices as partially ordered sets and their properties, lattices as algebraic system, Sub lattices, direct products and Homomorphisms of Lattices some special lattices eg Complete lattices, complemented lattices and distributive lattices.

References :

1. I. N. Herstein :- Topics in Algebra.
2. M.Artin :- Algebra
3. L. S Luther & I.B.S Passi :- Algebra Vols I & II Narosa Publication House
4. D.S. Dummit and R.M. Foote:- Abstract Algebra
5. N.S. Gopalakrishnan :- University Algebra
6. N.Deo :- Graph Theory with Applications to Engineering and Computer Science.
7. K.H. Rosen :- Discrete Mathematics and its applications.
8. S. Lipschutz and M. Lipson :- Discrete Mathematics
9. C. L. Lin:- Elements of Discrete Mathematics.

PAPER II (M-102)

Real Analysis I and Measure Theory I

Real Analysis I

Unit 1: Sequences and series of functions, pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass-M test, Abel's and Dirichlet's test for uniform convergence**.**

Unit 2: Uniform convergence and differentiation, Weierstrass approximation theorem Power series, Uniqueness theorem for power series, Able's and Tauber's theorem.

Measure Theory I

Unit 3: Lebesgue outer measure, Measurable sets Measuribility, Measurable functions, Borel and Lebesque measurability, non- measurable sets.

Unit 4 : Integration of non-negative functions, the general integral, Integration of series, Riemann and Lebesgue integrals.

Unit 5 : The Four Derivatives, function of bounded variation, Lebesgue differentiation Theorems, Differentiation and Integration.

References :

1. W. Rudin :- Principles of Mathematical Analysis
2. T. M. Apostal :- Mathematical Analysis
3. I.P. Natanson :- Theory of function of Real Variable
4. H.L. Royden :- Real Analysis
5. G.de Barra :- Measure Theory and Integration
6. P.K. Jain and V.P Gupta :- Lebesque Measure and Integration
7. I.K. Rana :- An Introduction to Measure and Integration
8. P.R. Halmos- Measure Theory.

PAPER III (M-103)

Fuzzy Set Theory and Topology I

Fuzzy Set Theory:

Unit 1 : Fuzzy Sets Versus Crisp sets, Basic definitions, types, properties and representations of Fuzzy sets, Convex Fuzzy sets, Basics operation on Fuzzy set, α- Cuts, Decompositions theorem, Complements, t- norm and t-conorms, Exten -sion principles and Simple applications of Fuzzy sets.

Topology I

Unit 2 : Definition and examples of topological spaces, closed sets, dense subsets, Neighbourhood, interior, exterior, boundary and accumulation points. Derived Sets, metrizable and non-metrizable topological spaces, Hausdorff spaces, Bases and subbases. Subspaces and Relative topology. Product of afinite number of topological spaces. Alternative method of defining a topology in terms of Kuratowski closure operator and neighbourhood system.

Unit 3 : Continuous functions and homeomorphism, characterisation of continuity in Terms of open sets, closed sets, basic open sets, sub- basic open sets and closure. First and second countable topological spaces Lindelof's theorem, separable Spaces, second countability and separability**.**

Unit 4 : Separation axioms T_0, T_1 and T_2 and their basicsproperties, compactness, Continuous function and compact sets, basic properties of compactness and Finite intersection property**.**

Unit 5 : Connectedness, continuous function and connected sets characterization of Connectedness in terms of a discrete two point space connectedness on real line.

References :

1. G.F. Simmons :- Introduction to Topology and Modern Analysis
2. J.R. Munkres :- Topology, K.K.Jha :- Advanced General Topology
3. Pundir And Pundir:- Fuzzy Sets & their Application, G.J.Klir & B. Yuan :- Fuzzy sets

PAPER IV (M-104)

Differential and Integral Equations and Complex Analysis

Differential and Integral Equations

Unit 1: Initial Value problem and the equivalent integral equation, n order equation in d dimension as a first order system. Concepts of local existance, existance in the large and uniqueness of solution with examples. Integral Equations and their classifications. Eigen values and eigen functions. Fredholm Integral equations of Second Kind, Iterative Scheme and method of successive approximations.

Unit 2 : Ascoli- Arzela theorem, a theorem on convergence of solutions of a family of Initial value problems. Picard- Lindelof theorem, Peano's existance theorem Corollaries, Kamke's convergence theorem.

Unit 3 : Gronwall's inequality, maximal and minimal solution, Differential inequalities, Uniqueness theorem, Nagumo's and Osgood's criteria, successive approximations.

Complex Analysis

Unit 4 : Taylor's theorem, Maximum modulus Principle, Schwarz's Lemma, Laurent Series, Isolated singularities, Meromorphic function, Mittag-Leffler's theorem The argument principle, Rouche's theorem, power series**.**

Unit 5 : Residues, Cauchy's residue theorem, Evaluation of integral, Branches of many valued functions with special reference to argz, logz and Z^n , Bilinear transformations, their properties and classifications, definition and examples of conformal mappings.

References :

1. P. Hartman :- Ordinary Differential Equation
2. J.B. Conway :-Functions of one Complex Variables,
3. L.V. Ahlfors :- Complex Analysis
4. S.G.Mikhlin :- Linear Integral Equations.
5. R.P.Kanwal :- Linear Integral Equations, Theory and Techniques.
6. S. Ponnusamy :- Foundations of Complex Analysis**.**

Syllabus of M.A/M.Sc (Mathematics) Semester II

PAPER V (M-201)

Abstract Algebra – II and Linear Algebra

Abstract Algebra II

Unit 1 : Cyclic Modules, simple Modules, semi-simple Modules, Schur's Lemma, Free Modules**.**

Unit 2 : Solution of equations by radicals, insolvability of equations of degree 5 by radicals.

Unit 3 : Similarity of linear transformations, Invariant subspaces, reduction to triangular forms, Nilpotent transformations, Index of Nilpotency, invariants of a Nilpotent transformations, primary decomposition theorem, Joardan blocks and Jordan forms rational canonical form.

Linear Algebra

Unit 4 : Bilinear form, algebra of bilinear form Matrix of bilinear forms, degenerate and Non-degenerate bilinear forms, Alternating bilinear forms

Unit 5: Symmetric and Skew-symmetric bilinear forms, Quadratic form, law of Inertia, Sylvester's theorem, Hermitian forms.

References :

1. Same as references 1-4 of Paper I of Semester I
2. K.B.Datta:- Matrix and Linear Algebra
3. S. Lipschutz:- Linear Algebra, Schaum's outline series
4. Hoffman and Kunze:- Linear Algebra

PAPER VI (M-202)

Real Analysis II and Measure Theory II

Real Analysis II

Unit 1 : Definition and examples of Riemann-Stieltje's integral Property of integral, Integration and differentiation, the fundamental theorem of Calculus, Integration Of vector valued function, rectifiable curves.

Unit 2 : Functions of several variables, linear transformation, Derivatives in an open subset of R^n,chain rule,partial derivatives, interchange of order of differentiation, derivatives of higher orders, Taylor's theorem.

Unit 3 : Inverse function theorem, Implicit function theorem, Jacobians, Extremum Problems with constraints, Lagrange's multiplier methods, differentiation of Integrals, partition of unity, Differential forms, Stoke's theorem**.**

Measure Theory II

Unit 4 : Measure and outer measure, extension of measures,uniqueness of extension, Completion of a measure, measurable spaces, Integration with respect to a measure.

Unit 5 : The L^P-spaces, convex functions, Jensen inequality Holder's and Minkowski's Inequalities, completeness of L^P-spaces, convergence in measure, Almost uniform Convergence.

References :

Same as references of Paper II of Semester I

PAPER VII (M-203)

Topology II and Boolean Algebra

Topology II

Unit 1 : Regular and Normal spaces, T_3 and T_4 spaces, characterisations and basic properties, Urysohn's lemma and Tietze extension Theorems.

Unit 2 : Sequentially and countably compact sets, local compactness and one point Compactifications, Stone-Cech compactification, compactness in mertic spaces, totally bounded metric spaces, Lebesgue covering lemma, equivalence of compactness and sequential compactness in metric spaces**.**

Unit 3 : Components of a spaces, totally disconnected spaces and locally connected spaces and their important results, Tychonoff product topology in terms of standard sub-base and its characterisations, projections maps, Hausdorffness and product spaces, Compactness and product spaces,(Tychonoff's theorem), connectedness and product Spaces, Embedding and metrization, Embedding lemma and Tychonoff embedding, The Urysohn metrization theorem.

Boolean Algebra

Unit 4 : Boolean algebra as a complemented distributive lattice, Boolean rings, identification of Boolean algebra and Boolean rings, sub-algebra and generators.

Unit 5 : Boolean homomorphism and ring homomorphism ideals in a Boolean algebra and Dual ideals, Fundamental theorem of homomorphism and Stone's representation theorem for Boolean algebras and Boolean rings, simple application to eletrical network, solvability of Boolean equations and logical puzzles.

References

1. Same as references 1 to 3 of paper III of Semester I
2. E.Mendelson :- Boolean Algebra and Switching Circuits
3. Kolman, Bushi and Ross :- Discrete Mathematical Structure.

PAPER VIII (M-204)

Integral Transform and Differential Geometry

Integral Transform

Unit 1 : Laplace transform, Elementary properties including convolution. The Bromwich Integral for Laplace transform, simple examples and applications to ordinary linear differental equations with constant coefficients.

Unit 2 : Fourier transform, Fourier Integral theorem, Fourier sine and cosine transforms (finite and infinite) simple examples and applications to ordinary linear differential equations with constant coefficients.

Differential Geometry

Unit 3 : Curves in R^3 spaces, parameters other than arc lengths, tangent principal normal, binormal and three fundamental planes, Curvature and torsion of space curves, Serret- Frenet formulae, Fundamental theorem on spaces curves, Helices, spherical. indicatrix, Involutes and Evolutes, Bertrand curves.

Unit 4 : Representation of surfaces, Curves on surfaces in R^3 spaces, tangent plane and Normal, Envelope, characteristic and edge of regression, developable surface of revolution, directions on a surface, paramertric curves, angle between them, first order and second order magnitudes, principal directions and lines of curvature, Normal Curvature, Euler's theorem and Meunier's theorem.

Unit 5 : Conjugate directions, Isometric lines, asymptotic lines and Geodesics- their equations and properties, curvatrure and torsion, their structures on surfaces of revolution, Bonnet's theorem, Clairaut's theorem and Dupin's indicatrix.

References

1. R.V. Churchill ;- Operational Mathematics
2. Vasistha and Gupta :- Integral Transform
3. C.E. Weatherburn:- Differential Geometry In Three Dimension
4. J.A. Thorpe :- Elementary Topics in Differential Geometry.

Syllabus of M.A/M.Sc (Mathematics) Semester III

PAPER IX (M-301)

Functional Analysis I, Advanced Topology I and Algebraic Topology

Functional Analysis I

Unit 1 : Normed linear spaces, Banach spaces and examples, Quotient space of normed linear Spaces and its completeness, equivalent norms, Riesz Lemma, Basic properties of finite dimensional normed linear spaces and compactness.

Unit 2 : Weak convergence and bounded linear transformation, normed linear spaces of bounded linear transformations, dual spaces with examples, uniform boundness theorem and some of its consequences.

Unit 3 : Open mapping theorem and closed graph theorem, Hahn- Banach Theorem on real linear spaces, complex linear spaces and normed linear spaces, Reflexive spaces.

Advanced Topology I

Unit 4 : Directed Sets, Nets and Subnets, convergence of nets, characterization of Hausdorffness and continuity in terms of nets, characterization of compactness in terms of nets, nets in topological products

Algebraic Topology

Unit 5 : Homotopy, Homotopy of paths, Homotopy class, Loop Homotopic, path connected, simply connected, Fundamental group, Fundamental group of a circle Covering maps and spaces, Covering Homotopy Lemma and Fundamental theorem of algebra**.**

References

1. G.F.Simmons:- Introduction to Topology and Modern Analysis
2. K.K.Jha :– Functional Analysis, Advanced General Topology
3. Futton:- Algebraic Topology First Course

PAPER X (M-302)

Mathematical Methods

Unit 1 : Orthogonalisation, Bessel's Inequality, Mean error minimization,completeness relation, Weierstrass approximation theorem, polynomials of Legendre, Hermite and Bessel, generating function, orthogonality, recurrence relation and Rodrigue's formula

Unit 2 : Partial Differential Equation and properties, concept of well posed problems, Reduction of P.D.E in two independent variables to the canonical forms, classification in to elliptic, hyperbolic and parabolic equations, Laplace's equations in cartesian, cylindrical and spherical co-ordinates, Equipotential surfaces, Interior and exterior Dirichlet problem, the Maximum- Minimum property, solutions and Uniqueness, Dirichlet's problem for a circle, fundamental properties of Harmonic function.

Unit 3 : Wave equation in one dimension and two dimension, vibrations of struck and plucked string with fixed ends, homogeneous rectangular and circular membranes, eigen vibrations, D'Alembert's solution of one dimensional wave equation. One dime -nsional Diffusion equation & solution of initial value problem by integral transform .

Unit 4 : Tensors- Transformations of Co-ordinates, contravariant and covariant vectors Symmetric and skew-symmetric tensors, addition and multiplication of tensors, Contraction and composition of tensors, Quotient law.

Unit 5 : Reciprocal symmetric tensors of the second order, Christoffel's symbols, covariant derivative of a contravariant vector, Co-variant derivative of a covariant vector, covariant derivatives of tensors, curl of a vector, Divergence of a covariant vector, Laplacian of a scalar invariant.

References

1. I. N. Sneddon:- Elements of Partial Differential Equations
2. R. Courant and D. Hilbert:- Methods of Mathematical Physics Vol I & Vol II
3. C.E. Weatherburn : - Riemannian Geometry and Tensor calculus
4. Smirnov and Tychonoff : - Partial Differential Equations.

PAPER XI (M-303)

Analytical Dynamics I and Fluid Mechanics I

Analytical Dynamics I

Unit 1 : Generalised Co-ordinates, Holonomic and Non Holonomic systems, Lagrange's equations of motion, energy equations for conservative fields**.**

Unit 2 : Hamilton's canonical equations, Rouths equations, Hamilton's Principle, Principle of Least Action**.**

Unit 3 : Small Oscillations, normal Co-ordinates, normal mode of vibration.

Fluid Mechanics I

Unit 4 : Lagrangian and Eulerian methods, Equation of Continuity, Boundary Surfaces, Stream lines, Path lines and Streak lines, velocity potential, irrotational and rotational motions, vortex lines.

Unit 5 : Lagrange's and Euler's equations of motion, Bernoulli's theorem, equation of motion by flux method, equation referred to moving axis, implusive actions.

References

1. A.S. Ramsey :- Dynamics Part II
2. S.L. Loney :- Dynamics of particle and rigid bodies
3. F.Chorlton :- A text Book of Fluid Dynamics.
4. M.D. Raisinghania:- Fluid Dynamics

PAPER XII (M-304)

Optimization Techniques (Optional)

Linear Programming

Unit 1 : Simplex method for unrestricted variable, Two phase method, Dual simplex method, Parametric Linear programming, Upper Bound technique, Interior point algorithm, Linear Goal programming.

Unit 2 : Integer programing, Branch and bound technique, Gomory's algorithm.

Non- Linear programming :

Unit 3 : One and multi-variable unconstrained optimization, Kuhn- Tucker condition for constrained optimization, Wolfe's and Beale's methods.

Operation Research

Unit 4 : Game theory, Two person- Zero sum games with mixed strategies, Graphical solution by expressing as a linear programming problem.

Unit 5 : Inventory theory, Different costs of inventory model, Deterministic Economic lot size model, EOQ with uniform demand and several productions of unequal length / production runs of equal length EOQ models- Shortages not allowed, shortages allowed.

References

1. H.A.Taha :- Operations Research- An Introduction
2. Kanti Swarup, P.K.Gupta and Man Mohan: Operations Research
3. P.K.Gupta and D.S. Hira :- Operations Research- An Introduction

PAPER XIII (M-401)

Functional Analysis II and Advanced Topology II

Functional Analysis II

Unit 1: Inner product spaces, Riesz lemma on Hilbert space, orthonormals sets and Parseval's identity, structure of Hilbert spaces, Projection theorem Riesz Representation Theorem.

Unit 2 : Adjoint of an operator on a Hilbert space, Reflexivity of Hilbert spaces, Self-adjoint Operators, positive operator, Projection, Normal and unitary operators.

Advanced General Topology II

Unit 3: Filters and Ultra Filters, Filter base, Image filter and induced filter, convergence of Filters.

Unit 4 : Characterization of Hausdorffness in terms of filters, Characterization of continuity in terms of filters, Characterization of compactness in terms of filters, Tychonoff theorem on product of Compact spaces.

Unit 5 : Approximation theory- the function algebras C (X, R) and C (X, C), the Weierstrass approximation theorem, Stone-Weierstrass theorem for real and Complex cases, Extended Stone-Weierstrass theorems**.**

References

Same as references 1 and 2 of paper IX of semester III.

PAPER XIV (M-402)

Integration Theory and Topological Vector Spaces

Integration Theory

Unit 1: Signed Measure, Hahn decomposition theorem, mutually singular measures, Radon-Nikodym theorem, Lebesgue decomposition theorem, Riesz Representation Theorem.

Unit 2 : Extension Theorem (Cartheodory) , Lebesgue Stieltjes integral, Product measures, Fubini's theorem, Differentiation and integration, Decomposition into absolutely Continuous parts.

Unit 3 : Baire sets, Baire Measure, Continuous functions with compact support, Regularity of measure on locally compact spaces**.**

Unit 4 : Integration of continuous function with continuous support, Reisz- Markoff Theorem.

Topological Vector Spaces (TVS)

Unit 5 : TVS , Notion of local base, balanced, absorbing and bounded set in TVS, Convex sets, Convex Hull and semi-norms in TVS, Locally Convex Spaces(LCS) Examples and simple results related to TVS and LCS**.**

References

1. Same as References of 5 to 8 of Semester I
2. A.E. Taylor- Introduction to Functional Analysis

PAPER XV (M-403)

Analytical Dynamics II and Fluid Mechanics II

Analytical Dynamics II

Unit 1 : Contact transformations, Lagrange brackets and Poisson brackets, the most general infinitesimal contact transformation, Hamilton- Jacobi equation.

Unit 2 : Motivating problem of Calculus of variation, Euler- Lagrange equation shortest distance, minimum surfaces of revolution, Brachistochrone problem.

Fluid Mechanics II

Unit 3 : Irrotational Motion in two dimension, stream function, complex velocity potential, sources, sinks, doublets and their images, conformal mapping, Milne-Thompson circle theorem**.**

Unit 4 : Two dimensional irrotational motion produced by motion of a circular, coaxial and elliptic cylinders in an infinite mass of liquid, kinetic energy of a liquid, Theorem of Blasius, motion of a sphere through a liquid at rest at infinity, liquid streaming past a fixed sphere, Equation of motion of a sphere, Stoke's stream function

Unit 5 : Vortex motion and its elementary properties, Kelvin's proof of permanence, Motion due to circular and rectilinear vertices**.**

References

Same as references in Paper XII of Semester III.

PAPER XVI (M-404)

Operations Research (Elective Paper)

Unit 1: Queuing Theory- Poisson probability law, Distribution of inter-arrival time, Distribution of time between successive arrivals, Differential difference equation of M |M |1 : ∞ |FIFO , M |M |1 : N|FIFO , M |M |C : ∞ |FIFO , M |M |C : N|FIFO ,

Unit 2: Information Theory: Description of communication system, Mathematical definition of information, Axiomatic approach to information, Measures of uncertainty, Entropy In two dimensions- property, conditional entropy.

Unit 3 : Channel capacity, Efficiency and redundancy, Encoding, Fano-encoding procedure, Necessary and sufficient condition, average length of encoded message.

Unit 4: Replacement Model- introduction concepts of present value, replacement of items whose maintenance cost increase with time and value of money also changes, Replacement of items that fail completely, individual and group replacement policy.

Unit 5 : Sequencing – N jobs and 2 machines, N jobs and 3 machines, N jobs M machines.

References

Same references as in paper XII of semester III.

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